

# A Condition Monitoring Vector Database Approach for Broken Bar Fault Diagnostics of Induction Machines

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**Abstract** – In this paper, a Condition Monitoring Vector Database (CMVDB) approach for broken bar fault diagnostics of squirrel-cage induction machines is presented. In this approach, a database of so-called “condition monitoring vectors” (CMVs) is generated for healthy and broken bar fault conditions using the time-stepping Finite-Element method. The CMV consists of the negative sequence components of winding voltages, currents, and impedances, the frequency spectrum sideband components of motor currents, and the space-vectors of motor terminal quantities (currents and voltages) from which the motor magnetic field pendulous oscillations are derived, as well as the motor speed and developed torque. This CMV will serve as the fault index (signature) for the faults under investigation in this work. This database is intended for use as a reference database in an on-line condition monitoring and fault diagnostic system. In this work, Artificial Intelligence (AI) techniques based on a statistical machine learning approach are used to detect and distinguish the type of fault and its severity based on the on-line measurements of the motor terminal voltages and currents, as well as the motor speed and developed torque, in comparison to the available CMVDB. To demonstrate the proof-of-principle of the database approach, simulation and experimental results for a 2-hp induction motor are given here to verify the viability of this approach.

**Keywords** – Induction motor, fault diagnostics, condition monitoring, artificial intelligence, database.

## I. INTRODUCTION

In the past two decades, there have been many investigations on condition monitoring and fault diagnostics in electric machines, especially squirrel-cage induction motors such as in [1-17]. In many applications involving medium to large integral horsepower motors, it is worthwhile to perform sophisticated fault simulations encompassing a large number of fault scenarios and to verify some of these scenarios experimentally. Thus, one can verify the validity of a simulation-generated database of fault signatures for use in machine condition monitoring and fault diagnostics. Such an elaborate analysis can be justified because sudden machine failure is very damaging or catastrophic in applications such

as large industrial systems or central station power plant auxiliaries in which the electric machine is the prime mover. Consequently, these machine failures reduce productivity in industrial and power systems. Hence, maintenance schedules can proactively be implemented to reduce or prevent these failures. Nevertheless, the probability of a sudden machine failure cannot be entirely ruled out. Moreover, increasing the frequency of scheduled maintenance increases the cost and decreases the productivity of a system. Accordingly, an on-line fault diagnostic system becomes a valuable tool to increase system efficiency and reliability.

Numerous diagnostic techniques for induction motors have been reported in the literature to diagnose electric machine faults, such as stator winding inter-turn shorts, broken rotor bars, broken end-ring connectors, and bearing faults. These diagnostic techniques include computation of the negative sequence components of motor terminal quantities [2-4], detection of the frequency spectrum sideband components [5,6], motor parameter estimation methods [7,8], Artificial Intelligence (AI)-based statistical machine learning approach [9,10], wavelet transformation techniques [11], artificial neural networks [12-14], as well as the recently proposed motor magnetic field pendulous oscillation phenomenon [15-17]. However, the use of such individual techniques alone is sufficient to detect only certain types of faults. In situations where mechanical load variations, partial loading, unbalanced utility voltage supply, or vector-control drive excitations are concerned, some of the aforementioned techniques, if used in a stand-alone manner, may not be reliable enough for fault diagnostics, and may yield a false indication of fault where non exists. Or worst, a fault could be masked by compensation action taken by the drive’s control system.

The combination of some of the techniques mentioned above led the present investigators to another form of broken bar fault diagnostic approach. The approach presented in this paper is based on a Condition Monitoring Vector Database (CMVDB). This CMVDB consists of sets of so-called “condition monitoring vectors” (CMVs) for healthy and broken bar fault conditions. This CMV consists of negative sequence components of motor voltages, currents and impedances, frequency spectrum sideband components of motor currents, and space-vectors of the motor terminal quantities (currents and voltages) from which motor magnetic field pendulous oscillations are obtained, in addition to motor speed and developed torque. One knows that the negative

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sequence components of the motor quantities will not be affected by the broken bar faults since the faults will not introduce any asymmetry effects in the stator windings. However, the inclusions of these negative sequence components in the CMV is to allow the flexibility of detecting any stator-winding inter-turn shorts which will be carried out in the near future.

In this work, the database can best be generated using the time-stepping Finite-Element (TSFE) method. The generated healthy and faulty CMV database is used as a reference database for on-line condition monitoring and fault diagnostic purposes. Meanwhile, an AI-based statistical machine learning approach is developed to detect and distinguish (or characterize) the type of fault and its severity, based on the on-line measurements of the motor terminal currents and voltages, as well as the motor speed and developed torque, in comparison to the CMVs available in the CMVDB system. This CMVDB approach is useful in detecting a plethora of faults. Since the TSFE method is a time-consuming process, which involves complex computation algorithms, the CMVDB approach will be worthwhile and appropriate for applications involving substantial integral or large horsepower induction machines, such as in large industrial and power systems. For purposes of this proof-of-concept investigation, the CMVDB approach is verified using a 2-hp induction motor, which was TSFE simulated and experimentally tested in the laboratory under healthy and various faulty operating conditions.

## II. CONDITION MONITORING VECTOR DATABASE APPROACH

The CMVDB approach consists of two parts, namely (1) the TSFE simulations, which compute the machine performance such as the machine winding currents and voltages, and (2) the CMV computations, which calculate the required indices used for fault diagnostics.

### A. TSFE Simulations

The time-stepping Finite Element (TSFE) technique computes on a time instant-by-instant basis (time profiles/waveforms) the input phase and line currents, voltages, developed power, and torque of a motor as functions of the particular magnetic circuit, winding layouts, and materials subject to the given inverter (power conditioner) topologies and operating conditions. Computations include the full effects of interaction of machine space harmonics with time-domain harmonics due to modern fast electronics switching on overall motor-controller/drive performance [18-21]. In addition, the TSFE algorithms can also be used in parametric design studies.

The TSFE aspect fully incorporates the nonlinear effects of magnetic saturation in the machine and makes full use of the natural machine winding's frame of reference. Hence, this

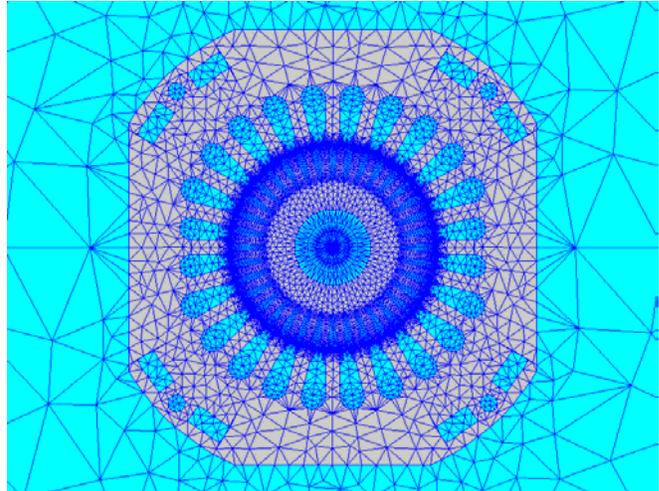


Fig. 1. FE grids of the 2-hp induction motor.

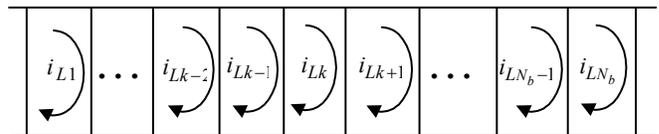


Fig. 2. Schematic representation of the modeling of a healthy cage.

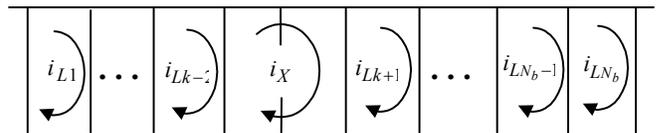


Fig. 3. Schematic representation of the modeling of a broken rotor cage bar.

assures inclusion of all significant space harmonics due to the physical design and nonlinear nature of the motor's magnetic circuits, as well as the time harmonics generated from the inverter switching in the motor-drive modeling and simulations. The TSFE method is applied here on a 230-volt, 60-Hz, 2-pole, 2-hp, three-phase squirrel-cage induction motor with 36 rotor cage bars and 24 stator slots, an FE grid of which is shown in Fig. 1.

Commercially available FE software packages, namely (1) Magsoft\* and (2) MagneForce\*, were used to simulate the 2-hp induction motor. In order to simulate the bar breakages, the squirrel-cage loops need to be disturbed according to the patterns shown in Fig. 2 (healthy) and Fig. 3 (one broken bar). In other words, in the FE model, a segment of the bar is replaced by a non-conductive material to emulate one or more broken bar cases. The TSFE simulation is carried out here under direct-line utility grid excitation condition. Upon generating the time-domain voltages and currents from the FE simulations for the healthy and broken bar fault conditions, the next step is to compute the CMV as described next.

\* (1) Magsoft Corporation – Flux 2D

\* (2) MagneForce Software Systems, Inc. – Transient and Standard Solver

### B. CMV Computations

In this part of the CMVDB process, the previously mentioned methods such as the negative sequence approach, the frequency spectrum sideband technique, and the pendulous oscillation phenomenon are used to generate a database of CMVs based on the FE computed time-domain voltages and currents under healthy and faulty conditions. To clearly illustrate the idea of generating a database of CMVs, an 8-dimensional CMV can be expressed as follows:

$$CMV = \left[ \frac{V_n}{V_p} \quad \frac{I_n}{I_p} \quad \frac{Z_n}{Z_p} \quad A(f_{LSB}) \quad \Delta\delta_{BB} \quad \Delta\delta_{SC} \quad \omega_m \quad T_{dev} \right]^T \quad (1)$$

Here, the first three terms of the CMV are the ratios of the negative and positive sequence components of the stator voltages,  $V$ , the stator currents,  $I$ , and the associated impedances,  $Z$ , where the  $n$  and  $p$  subscripts indicate the negative and positive sequence components, respectively. The first term of (1) is used to detect any unbalanced utility voltage power supply, whereas the second and third terms are used to detect stator winding inter-turn shorts [2-4] which will be carried out in the near future. It is of importance to point out that in symmetrical component theory that an unbalanced system will introduce negative sequence components [22,23]. Therefore, unbalanced systems result from either an unbalanced utility voltage power supply or stator winding inter-turn shorts, which will introduce negative sequence voltage and current components. Accordingly, using only the negative sequence current component alone for stator winding inter-turn fault detection is insufficient since an unbalanced voltage power supply will also introduce a negative sequence current component, and hence a false indication of a stator fault will be given under this condition.

Meanwhile, the fourth term of the CMV of (1) represents the normalized amplitude of the low sideband (LSB) frequency spectrum component of the stator current at the frequency,  $f_{LSB} = (1-2s)f_s$ , where  $f_s$  is the power supply frequency [5,6], relative to the amplitude of the fundamental frequency spectrum component of the stator current. This term is used for the detection of broken rotor cage bars [5,6]. The fifth and the sixth terms of the CMV of (1) are obtained using the motor magnetic field pendulous oscillation phenomenon [15-17] and they represent the range of oscillation (swing angle,  $\Delta\delta$ ) of the resultant mid air-gap magnetic field for broken rotor bars,  $\Delta\delta_{BB}$  [16], and the stator winding inter-turn faults,  $\Delta\delta_{SC}$  [17]. For more details on the concept of the pendulous oscillation technique, [15-17] should be consulted.

In order to calculate the slip and hence locate the low sideband frequency component for broken bar fault detection, the motor speed has to be acquired which represents the seventh term of (1). The eighth and final term of (1) is the developed motor torque and is used to help in the process of

fault detection due to the mechanical load variations under healthy and faulty operating conditions. It is of importance to mention here that the whole process of generating the database is accomplished in an a priori manner off-line, after which the database is further used for on-line condition monitoring and fault diagnostics. It should be pointed out that the CMV can be a  $k$ -dimensional vector, where  $k$  represents the number of required indices (signatures) used for fault diagnostics of various kinds. Here, in this work,  $k = 8$ . The advantage of this approach is that if one of the member indices of the CMV was unable to detect the presence of a fault, say the frequency spectrum sideband approach, another member of the CMV, say the pendulous oscillation swing angle may be able to detect such a fault with a higher degree of certainty. In addition, unbalanced voltage power supply or mechanical load variations which may cause a false indication of fault can be avoided because for example such a voltage unbalance will show up in the acquired physical on-site data.

### III. AI-BASED STATISTICAL MACHINE LEARNING APPROACH

An AI-based statistical machine learning approach is employed to train (learn) the TSFE simulated healthy and faulty CMVs using Gaussian Mixture Models (GMMs) approach [24] followed by the classification of the test (experimental) data based on the trained (learned) GMMs using a Bayesian maximum likelihood classifier [24]. The CMV computed from a set of current and voltage waveforms will only generate a single set of CMV, which will appear to be a single point in the  $k$ -dimensional space. To generate more data points for the training and classifying processes, a set of CMV is calculated for each complete ac cycle of the motor's current and voltage waveforms. Accordingly, the CMV of (1) will be expressed in matrix form of dimension  $k \times l$ , again where  $k$  represents the number of required indices (signatures) used for fault diagnostics, and  $l$  represents the number of complete ac cycles of the current and voltage waveforms. This means that there will be  $l$  data points in the  $k$ -dimensional space.

To categorize the TSFE simulated CMVs into different classes (healthy and faulty) using the GMMs approach, the algorithm must learn the GMM probability distribution for each class, which is defined as follows [24]:

$$p(x) = \sum_{m=1}^M w_m p_m(x) = \sum_{m=1}^M w_m N(x; \mu_m, \Sigma_m) \quad (2)$$

where,  $x$  represents a point in the  $k$ -dimensional space,  $M$  is the number of mixtures,  $N(x; \mu_m, \Sigma_m)$  is a normal distribution with mean,  $\mu_m$ , and covariance matrix,  $\Sigma_m$ , and  $w_m$  is the mixture weight having the constraint of  $\sum w_m = 1$ , [24]. The number of mixtures,  $M$ , can be determined based on the classification accuracy which tends to increase with the increased number of mixtures, provided there is sufficient

data to train on the GMM. The parameters for the GMM of (2) can be estimated using the well-known Expectation-Maximization (EM) method [25]. This iterative method yields a Maximum Likelihood estimate, through the estimation formulas, defined as follows:

$$\begin{aligned} \mu_m &= \frac{\sum_{j=1}^l p_m(x_j)x_j}{\sum_{j=1}^l p_m(x_j)}, \\ \Sigma_m &= \frac{\sum_{j=1}^l p_m(x_j)(x_j - \mu_m)^T(x_j - \mu_m)}{\sum_{j=1}^l p_m(x_j)}, \\ w_m &= \frac{\sum_{j=1}^l p_m(x_j)}{\sum_{j=1}^l \sum_{m=1}^M p_m(x_j)} \end{aligned} \quad (3)$$

After learning a GMM for each TSFE simulated CMV class (healthy and faulty), the next step is to classify the test data for healthy and faulty conditions based on the trained (learned) GMMs. This is done using a Bayesian maximum likelihood classifier, which computes the conditional likelihoods of the test data under each trained model and selects the model with the highest likelihood. The likelihoods are computed on a point-by-point basis from the trained models, as follows [24]:

$$p(\mathbf{X} | c_i) = \prod_{j=1}^l p(x_j | c_i) \quad (4)$$

where,  $\mathbf{X}$  is a CMV matrix of the test data,  $x_j$  is a point in the  $k$ -dimensional space, and  $p(x_j | c_i)$  is the probability of  $x_j$  given the  $i$ th class and is calculated using (2). The classification is:

$$\hat{c} = \arg \max_i p(\mathbf{X} | c_i) \quad (5)$$

where,  $\hat{c}$  is the maximum likelihood class. For further details on this AI-based machine learning approach, [24] should be consulted.

#### IV. TSFE SIMULATION RESULTS

The time-stepping Finite-Element (TSFE) simulation results of the 2-hp induction motor are presented here. They were carried out for the cases of healthy, one, three, and five broken bars. The motor is energized from a three-phase 230-volt source where the motor is parallel connected for that

level of voltage. For purpose of illustrations, a three-dimensional trajectory comprising three member indices of the CMV generated from TSFE simulations for the broken bar conditions is shown in Fig. 4. These three member indices are the swing angle,  $\Delta\delta_{BB}$ , the frequency low sideband component,  $A(f_{LSB})$ , and the ratio of the negative and positive sequence components of the stator currents,  $I_n/I_p$ .

One can observe the trajectory profile in terms of the  $\Delta\delta_{BB}$  and  $A(f_{LSB})$  as the number of broken bars increases. These TSFE simulated CMVs are introduced into the AI-based statistical machine learning environment from which the nature of these CMVs is trained and used for classification, as presented in the next section.

#### V. EXPERIMENTAL RESULTS

A 2-hp, 2-pole, 230-volt, three-phase induction motor was tested in the laboratory under broken bar fault conditions. A test setup of the induction motor system is shown in Fig. 5. The experimental CMVs of the 2-hp induction motor under direct-line utility grid excitation were obtained in a similar operating condition as the TSFE simulations, which are the healthy, one, three, and five broken bars. The CMVs were computed using the negative sequence concept, the frequency spectrum sideband technique, and the pendulous oscillation method as described in (1).

Again, for illustration purposes, a three-dimensional trajectory of the CMV from simulation and test results under healthy ( $H_{sim}$  and  $H_{test}$ ) and rotor broken bar (BB) fault conditions in the CMV coordinate frame of reference is shown in Fig. 6. Inspection of Fig. 6 reveals that the test and simulation results yield trajectories of similar trend and nature. Also, one can observe that the simulation and test results are in good agreement in terms of the swing angle,  $\Delta\delta_{BB}$ , and the frequency low sideband component,  $A(f_{LSB})$ , but

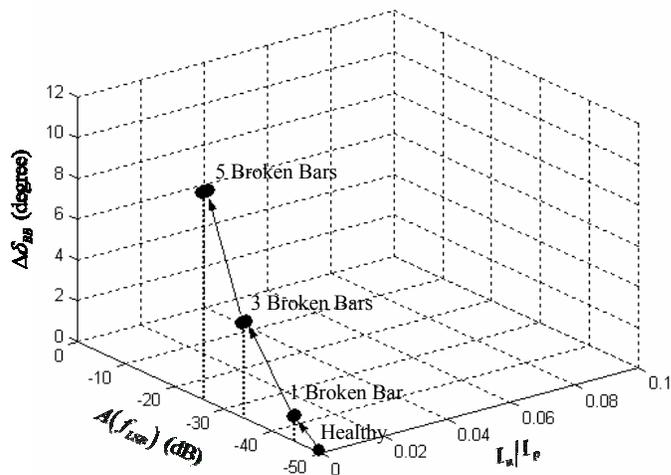


Fig. 4. Three-dimensional trajectory of the rotor broken bar faults in the CMV coordinate frame of reference.

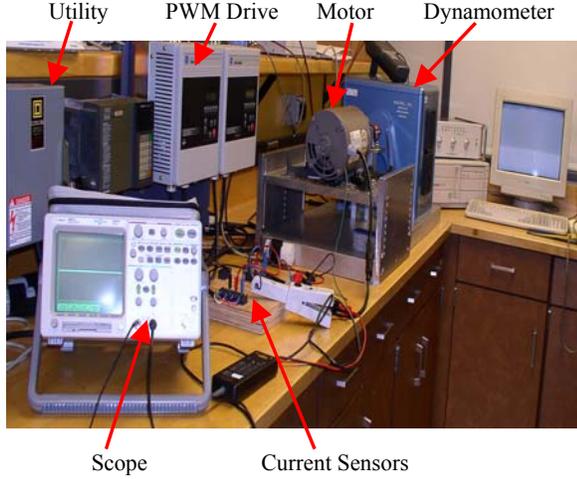


Fig. 5. Laboratory test setup.

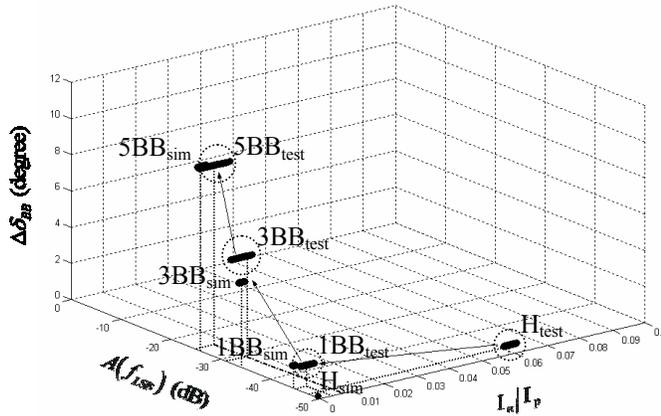


Fig. 6. Three-dimensional trajectory of the rotor broken bar faults in the CMV coordinate frame of reference (Simulation and Test).

not for the case of the ratio of negative and positive sequence components of the stator currents,  $I_n/I_p$ , under healthy condition. One would argue that the test results should have yielded an index  $I_n/I_p$  that should be zero for the rotor broken bar conditions, even for the healthy condition. But this is not the case here since this non-zero  $I_n/I_p$  is caused by the inherent motor winding structural imperfections which results in asymmetry of the winding currents. This is largely due to the nature of the “random type winding” of this class of motors.

As mentioned earlier, this paper develops an AI-based statistical machine learning technique to classify the type of fault and to predict its severity of the test data based on the trained models where the data were obtained from the TSFE simulations. The diagnostic results based on the probability distributions of the test data are given in Table I. As one can observe from Table I that the AI technique accurately detect and classify the type of fault by computing the highest proba-

TABLE I  
PROBABILITY DISTRIBUTIONS OF TEST DATA

Classes		Test Data			
		Healthy	1 BB	3 BB	5 BB
TSFE Training Data	Healthy	<b>0.725</b>	0.640	0.589	0.416
	1 BB	0.688	<b>0.989</b>	0.616	0.522
	3 BB	0.548	0.746	<b>0.945</b>	0.708
	5 BB	0.459	0.516	0.656	<b>0.997</b>

bility of each of the classes (healthy, one broken bar, three broken bars, and five broken bars) of the test data in correspondence to those of the training data.

## VI. CONCLUSIONS AND FUTURE WORK

An effective and robust diagnostic method for broken bar fault conditions has been introduced and examined in this paper for a 2-hp induction motor. This technique makes use of a Condition Monitoring Vector Database approach, which generates a database of condition monitoring vectors using time-stepping Finite-Element method. The TSFE simulation and test results show the distinct trajectory profile in the CMV coordinate of reference as the number of broken bars increases. Also, both the simulation and test results are in good agreement as far as the swing angle and the frequency low sideband components are concerned. In addition, the AI-based statistical machine learning approach is able to accurately detect and distinguish the broken bar faults and the severity.

Using a database for fault diagnostics is worthwhile and appropriate for integral and large horsepower electric motor-drive systems which enable a wide range of possible motor-drive fault scenarios to be modeled a priori to field deployment of the motor-drive system. Moreover, this CMVDB approach is able to detect different types of faults as long as the correct index is being used in the CMVDB. One of the possible future work is to detect stator winding interturn fault conditions using this proposed approach. Besides that, the work can be extended to fault prognostics in a sense that by monitoring the trajectory of the fault in the CMV coordinate reference frame, one can possibly predict the progression of the fault severity, and prognosticate as to when such severity might become intolerable for continuous operation.

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